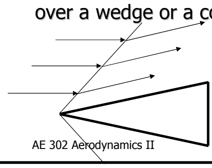
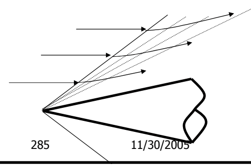
Supersonic Conical Flow

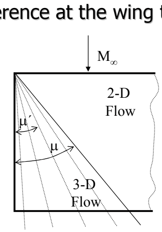
- In 2-D supersonic flow, shocks create abrupt changes in the flow which remain until another wave acts on it.
- In contrast, in 3-D or axi-symmetric supersonic flow, shock also exist, but flow properties may continue to vary after the shock.
- However, flow properties are constant along rays originating at the point of disturbance.
- This is best illustrated by the difference between flow over a wedge or a cone:

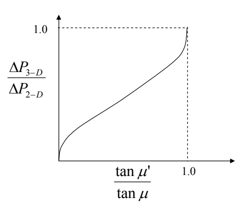




Supersonic Conical Flow [2]

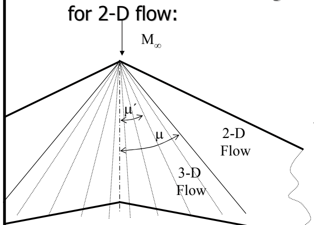
- Thus, in 3-D, we have what is knows as conical flow – flow defined by rays of constant properties.
- For example, if we consider the tip of a flat plate wing, a 3-D disturbance exists which leads to a zero pressure difference at the wing tip – just like subsonic flow.

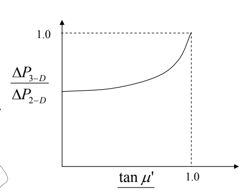




Supersonic Conical Flow [3]

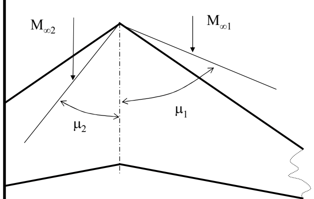
- A similar situation occurs at any leading edge break – like the apex of a swept wing.
- In this case, the pressure difference between upper and lower surface does not go to zero – but is below that for 2-D flow:

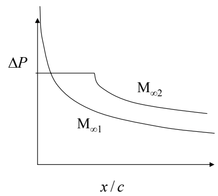




Supersonic Conical Flow [4]

- For such wings, the wing sweep can be high enough such that the leading edge is behind the apex shock wave – a so-called subsonic leading edge.
- In this case, the leading edge pressure difference becomes infinite – just like a flat plate in subsonic flow:





Supersonic Conical Flow [5]

- A supersonic leading edge requires sharp, thin wings in order to reduce shock strength.
- The F-104 fighter (max speed M=2.2) is a good example of a supersonic leading edge design.

Supersonic Conical Flow [6]

- The Concorde SST (max speed M=2.04) is a good example of a subsonic leading edge design.
- Note the complex loft of the wing which aligns the local wing to the upstream flow.

Supersonic Conical Flow [7]

- A wing can also have a subsonic trailing edge – when the trailing edge sweep is less then the flow Mach angle.
- For a subsonic trailing edge, the difference in pressure goes to zero, as the Kutta condition would predict.

Supersonic Panel Method

- To analyze supersonic flow, we can use methods which have corollaries to subsonic potential flow.
- To model wing thickness, we could use sources or doublets. In supersonic flow, these are : $\beta = \sqrt{M_\infty^2 - 1}$

$$\phi_s = -\frac{Q}{r_c} \quad \text{Source} \qquad \phi_d = \frac{\Lambda z \beta^2}{r_c^3} \quad \text{Doublet}$$

- This differ from our 2-D subsonic flow not only due to the 3rd dimension, but also due to the use of the "hyperbolic" radius:

$$r_c = \sqrt{(x - x_0)^2 - \beta^2\left[(y - y_0)^2 + (z - z_0)^2\right]}$$

- Here, the zero subscript indicates the location of the source/doublet.

Supersonic Panel Method [2]

- The hyperbolic radius has the property that it is imaginary for points outside a Mach cone originating from the disturbance.
- Since only real values are of interest, this means the disturbance will only affect the flow within the cone – as is expected for supersonic flow.
- In subsonic flow, we didn't need to model the wing thickness since we could determine the lift and induced drag without it.
- In supersonic flow, we might want to model the thickness in order to determine the wave drag due to thickness.

Supersonic Panel Method [3]

- However, in order to determine the lift, we will need some form of vortex element.
- In 3-D supersonic flow, a vortex can be defined by:

$$\phi_v = -\frac{\Gamma z v_c}{r_c} \quad \text{Vortex}$$

- Which includes a new factor given by:

$$v_c = \frac{x - x_0}{(y - y_0)^2 + (z - z_0)^2}$$

- This supersonic vortex element is the basis of the panel method solution used in the SPanel java applet on my web site.